Exercise 4

For each of the following integro-differential equations, classify as Fredholm, Volterra, or Volterra-Fredholm integro-equation

$$u'''(x) + u'(x) = x + \int_0^x tu(t)\,dt + \int_0^1 u(t)\,dt, \; u(0) = 0, \; u'(0) = 1, \; u''(0) = 1$$

Solution

There is both a Volterra integral (one with a variable limit of integration) and a Fredholm integral (one with constant limits of integration) in the equation, so it is a Volterra-Fredholm integro-differential equation.